Final Project

PSTAT122: Design and Analysis of Experiments

Winter 2026

STUDENT NAME

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Due Date

Due Date: Monday, March 16, 2026, 11:59 PM

1 Introduction

Objective: In this experiment, we will be examining the effects of different binary factors on the melting rate of ice. We will specifically be looking at the effect of container material, airflow, and light exposure on the time it takes to fully melt an ice cube using a 2^3 factorial experimental design with 8 total treatments.

Context and Motivation: Melting rate is determined by different heat transfer processes such as conduction, convection, and radiation, all of which are dependent on environmental conditions that influence how quickly heat is transferred to the ice. We chose a 2^3 factorial experimental design to measure the effect of each individual factor as well as their combinations, allowing us to test for both main effects and potential interactions between airflow, container material, and light exposure.

2 Experimental Design

We will be looking at 3 factors, each with two levels:

1. Airflow (high vs low)
    - A fan facing the ice cube will be on (high) or off (low)
    - We will use an Iris Woozoo fan
2. Container material (plastic vs metal)
3. Light exposure (high vs low)
    - An LED lamp facing the ice cube will be on (high) or off (low)

We will be measuring the response (melting time, in minutes) with an iPhone 16 Pro timer.

In this design, all three factors are fixed. The levels are each specific binary conditions that our group chose to test. A random factor would be one sampled from a larger population of possible levels; however, in our case, we use three binary factors that are each controlled.

As stated before, this is a completely randomized design with 2^3 factorial structure.

We will reduce bias from outside environmental factors like temperature or wind by randomizing the order of treatments. This will be implemented by a random number generator that assigns numbers to each treatment. Replication will be implemented by running each treatment combination five times under the same conditions. This is an important step because it allows us to estimate the experimental error and achieve a more reliable estimate of each effect. Repeating each of the eight combinations five times gives us a total of $8 \times 5 = 40$ observations. Blocking is not required in our case because the room will be a consistent temperature throughout the duration of the experiment and the location and person that does the testing will remain the same.

3 Data Collection

- **Procedure**: This experiment was conducted in a room in the UCSB Multicultural Center over 4 days total (03/07, 03/09, 03/10, and 03/14). We spent roughly 4 hours each day and arrived around 1p.m. each time. Our first step was to determine the order of treatment combinations randomly. This is the code we used:

```r
set.seed(123) # for reproducibility

# factors (initializing them like this will ensure each combination occurs 5 times)
Light <- rep(c("Low","High"), each = 20)
Container <- rep(rep(c("Plastic","Metal"), each = 10), times = 2)
Airflow <- rep(c("Low","High"), each = 5, times = 4)

# treatment number
Treatment <- rep(1:8, each = 5)

# build dataset
experiment <- data.frame(
  Treatment,
  Airflow,
  Container,
  Light
)


random_order <- sample(nrow(experiment))

# randomize run order
experiment_randomized <- experiment[random_order, ]

# add run order column
experiment_randomized$RunOrder <- 1:nrow(experiment_randomized)

# reorder columns
experiment_randomized <- experiment_randomized[, c("RunOrder","Treatment","Airflow","Container","Light")]

experiment_randomized
```

```
   RunOrder Treatment Airflow Container Light
31        1         7     Low     Metal  High
15        2         3     Low     Metal   Low
14        3         3     Low     Metal   Low
3         4         1     Low   Plastic   Low
38        5         8    High     Metal  High
25        6         5     Low   Plastic  High
26        7         6    High   Plastic  High
27        8         6    High   Plastic  High
32        9         7     Low     Metal  High
```

```
5         10         1     Low   Plastic   Low
19        11         4    High     Metal   Low
33        12         7     Low     Metal  High
35        13         7     Low     Metal  High
28        14         6    High   Plastic  High
9         15         2    High   Plastic   Low
37        16         8    High     Metal  High
8         17         2    High   Plastic   Low
7         18         2    High   Plastic   Low
10        19         2    High   Plastic   Low
34        20         7     Low     Metal  High
30        21         6    High   Plastic  High
4         22         1     Low   Plastic   Low
36        23         8    High     Metal  High
17        24         4    High     Metal   Low
11        25         3     Low     Metal   Low
23        26         5     Low   Plastic  High
40        27         8    High     Metal  High
12        28         3     Low     Metal   Low
22        29         5     Low   Plastic  High
39        30         8    High     Metal  High
21        31         5     Low   Plastic  High
13        32         3     Low     Metal   Low
2         33         1     Low   Plastic   Low
16        34         4    High     Metal   Low
29        35         6    High   Plastic  High
20        36         4    High     Metal   Low
1         37         1     Low   Plastic   Low
6         38         2    High   Plastic   Low
18        39         4    High     Metal   Low
24        40         5     Low   Plastic  High
```

We exported this table to a csv and created a column called "Time". We filled in the time, in minutes to two decimal places, in the order specified by the RunOrder column. This is our final table with the measured response included.

```r
data <- read.csv("fp_data_with_times.csv")
data
```

```
    X RunOrder Treatment Airflow Container Light  Time
1  31        1         7     Low     Metal  High 22.14
2  15        2         3     Low     Metal   Low 27.95
3  14        3         3     Low     Metal   Low 28.57
4   3        4         1     Low   Plastic   Low 37.12
5  38        5         8    High     Metal  High 12.68
6  25        6         5     Low   Plastic  High 31.06
7  26        7         6    High   Plastic  High 20.53
8  27        8         6    High   Plastic  High 21.12
9  32        9         7     Low     Metal  High 21.58
10  5       10         1     Low   Plastic   Low 35.68
11 19       11         4    High     Metal   Low 19.03
12 33       12         7     Low     Metal  High 22.87
13 35       13         7     Low     Metal  High 22.31
14 28       14         6    High   Plastic  High 19.88
15  9       15         2    High   Plastic   Low 25.47
16 37       16         8    High     Metal  High 11.96
```

```
17   8       17         2    High   Plastic   Low 26.31
18   7       18         2    High   Plastic   Low 24.88
19  10       19         2    High   Plastic   Low 24.79
20  34       20         7     Low     Metal  High 21.96
21  30       21         6    High   Plastic  High 20.29
22   4       22         1     Low   Plastic   Low 36.47
23  36       23         8    High     Metal  High 12.43
24  17       24         4    High     Metal   Low 17.89
25  11       25         3     Low     Metal   Low 28.36
26  23       26         5     Low   Plastic  High 29.95
27  40       27         8    High     Metal  High 12.19
28  12       28         3     Low     Metal   Low 27.81
29  22       29         5     Low   Plastic  High 31.28
30  39       30         8    High     Metal  High 11.71
31  21       31         5     Low   Plastic  High 30.62
32  13       32         3     Low     Metal   Low 29.14
33   2       33         1     Low   Plastic   Low 35.91
34  16       34         4    High     Metal   Low 18.74
35  29       35         6    High   Plastic  High 20.74
36  20       36         4    High     Metal   Low 18.42
37   1       37         1     Low   Plastic   Low 36.84
38   6       38         2    High   Plastic   Low 25.94
39  18       39         4    High     Metal   Low 18.21
40  24       40         5     Low   Plastic  High 30.41
```

The total time it took to measure the time of all 40 ice cubes melting was around 16 hours. This was significantly longer than we had initially expected, and it was challenging to build our schedules around the experiment time block each day. Aside from that, the process was very smooth and we did not run into any technical issues.

```r
library(dplyr)
data_with_calcs <- data %>% group_by(
  Treatment
) %>% summarize(
  Mean = mean(Time),
  StdDev = sd(Time),
  Min = min(Time),
  Max = max(Time)
)
data_with_calcs
```

```
# A tibble: 8 x 5
  Treatment  Mean StdDev   Min   Max
      <int> <dbl>  <dbl> <dbl> <dbl>
1         1  36.4  0.607  35.7  37.1
2         2  25.5  0.659  24.8  26.3
3         3  28.4  0.530  27.8  29.1
4         4  18.5  0.445  17.9  19.0
5         5  30.7  0.527  30.0  31.3
6         6  20.5  0.467  19.9  21.1
7         7  22.2  0.475  21.6  22.9
8         8  12.2  0.381  11.7  12.7
```

4 Analysis

- **Exploratory Data**: Here we first started with our analysis by looking at basic summary statistics and visualizations to better understand the data.

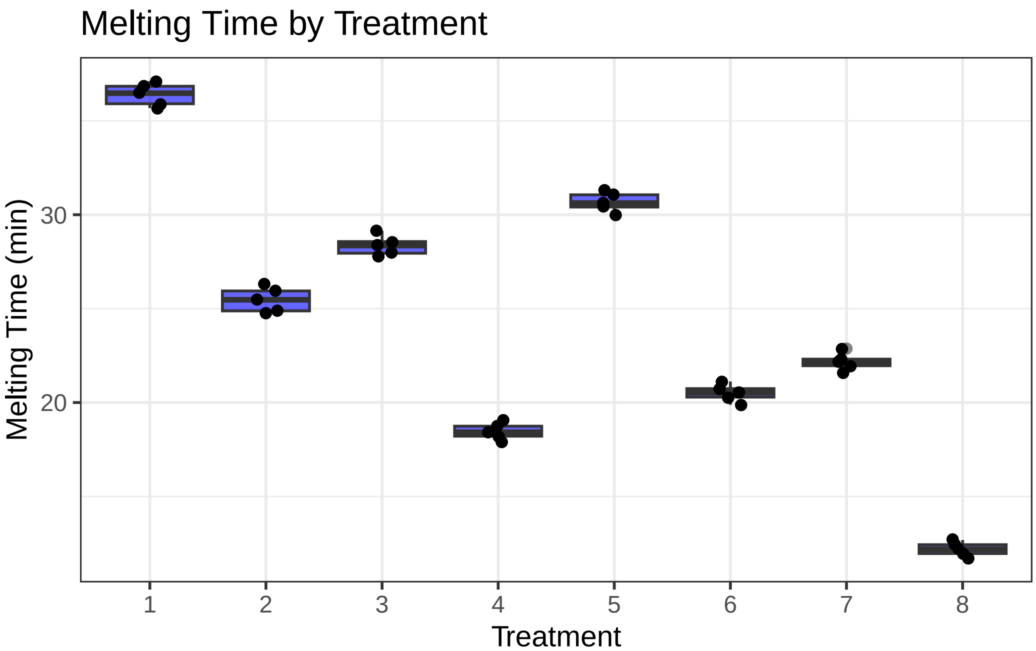
```
library(dplyr)

data <- read.csv("fp_data_with_times.csv")
data$Airflow <- factor(data$Airflow, levels = c("Low", "High"))
data$Container <- factor(data$Container, levels = c("Plastic", "Metal"))
data$Light <- factor(data$Light, levels = c("Low", "High"))

data %>%
  group_by(Treatment, Airflow, Container, Light) %>%
  summarize(Mean = round(mean(Time), 2),
            SD = round(sd(Time), 2),
            Min = round(min(Time), 2),
            Max = round(max(Time), 2),
            .groups = "drop")
```

```
# A tibble: 8 x 8
  Treatment Airflow Container Light  Mean    SD   Min   Max
      <int> <fct>   <fct>     <fct> <dbl> <dbl> <dbl> <dbl>
1         1 Low     Plastic   Low    36.4  0.61  35.7  37.1
2         2 High    Plastic   Low    25.5  0.66  24.8  26.3
3         3 Low     Metal     Low    28.4  0.53  27.8  29.1
4         4 High    Metal     Low    18.5  0.45  17.9  19.0
5         5 Low     Plastic   High   30.7  0.53  30.0  31.3
6         6 High    Plastic   High   20.5  0.47  19.9  21.1
7         7 Low     Metal     High   22.2  0.48  21.6  22.9
8         8 High    Metal     High   12.2  0.38  11.7  12.7
```

Initial Observations: From the summary statistics, it is clear that all three variables have some significant influence on the melting time, as melting quickly is associated with high airflow, the use of metal containers, and high light exposure. The treatment with the shortest melting time of 12.19 minutes was Treatment 8, which had high airflow, used metal containers, and had high light exposure. This was nearly three times faster than Treatment 1, which had low airflow, used plastic containers, and had low light exposure at 36.40 minutes. The standard deviations were small for all treatments, ranging from 0.38 to 0.66.

Boxplot by treatment

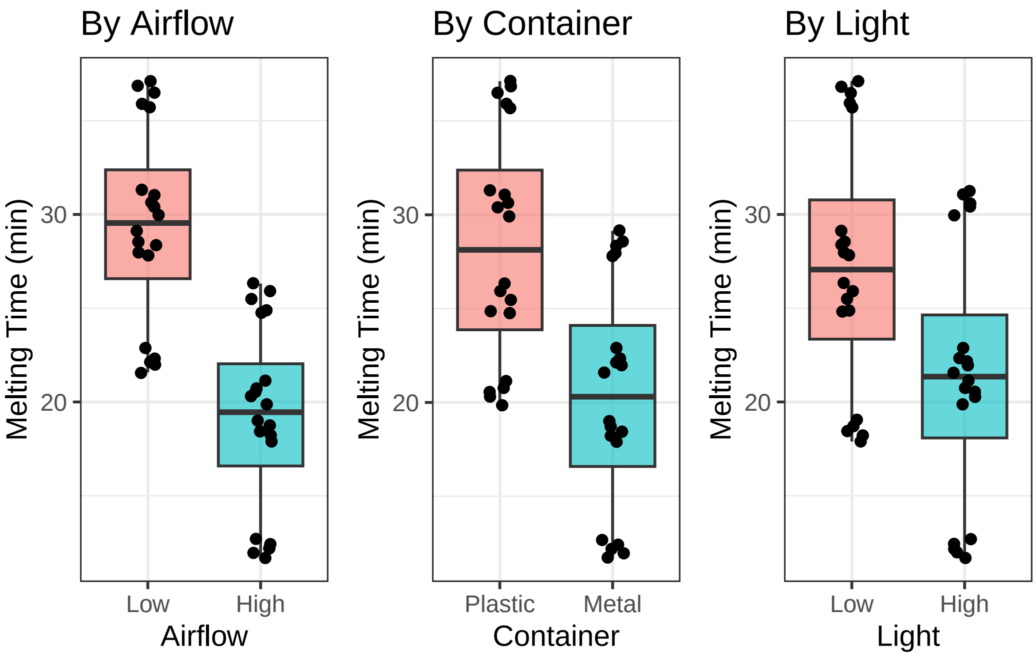
```
library(ggplot2)

ggplot(data, aes(x = factor(Treatment), y = Time)) +
  geom_boxplot(fill = "blue", alpha = 0.6) +
  geom_jitter(width = 0.1, size = 1.5) +
  labs(x = "Treatment", y = "Melting Time (min)", title = "Melting Time by Treatment") +
  theme_bw()
```



The box plot for each treatment shows that there is a distinct separation between all eight treatment groups with very little variation within each group, indicating that the factor combinations are having a strong and consistent effect on the melting time. The treatment with the longest melting time was Treatment 1 (low airflow, plastic, low light), while the shortest melting time was Treatment 8 (high airflow, metal, high light).

Boxplot for Each Factor

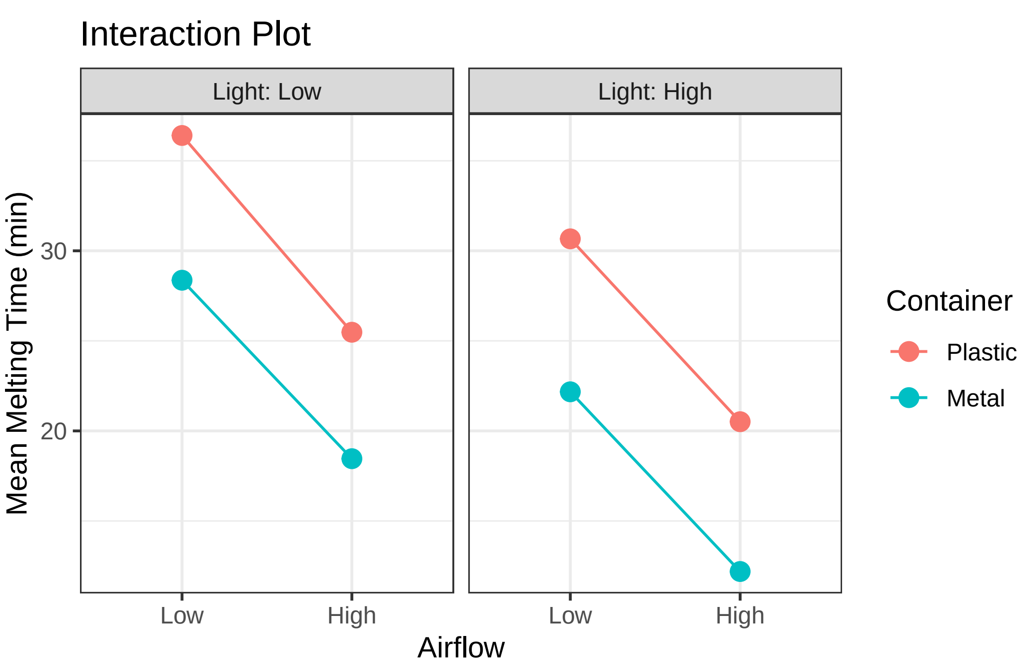
```
library(gridExtra)

p_air <- ggplot(data, aes(x = Airflow, y = Time, fill = Airflow)) +
  geom_boxplot(alpha = 0.6) + geom_jitter(width = 0.1, size = 1.5) +
  labs(y = "Melting Time (min)", title = "By Airflow") +
  theme_bw() + theme(legend.position = "none")

p_con <- ggplot(data, aes(x = Container, y = Time, fill = Container)) +
  geom_boxplot(alpha = 0.6) + geom_jitter(width = 0.1, size = 1.5) +
  labs(y = "Melting Time (min)", title = "By Container") +
  theme_bw() + theme(legend.position = "none")

p_lit <- ggplot(data, aes(x = Light, y = Time, fill = Light)) +
  geom_boxplot(alpha = 0.6) + geom_jitter(width = 0.1, size = 1.5) +
  labs(y = "Melting Time (min)", title = "By Light") +
  theme_bw() + theme(legend.position = "none")

grid.arrange(p_air, p_con, p_lit, nrow = 1)
```



The marginal box plots for each factor show that all three factors have a visible effect on melting time, with high levels of each factor being associated with faster melting times. Airflow has the largest effect, with high airflow being associated with a median melting time that is about 10 minutes shorter than low airflow, while container material has a similar but slightly smaller difference between melting times for plastic and metal containers. Light exposure has the smallest effect of the three factors, but the difference between low and high is still visible.

Interaction Plot

```
data %>%
  group_by(Airflow, Container, Light) %>%
  summarize(Mean = mean(Time), .groups = "drop") %>%
  ggplot(aes(x = Airflow, y = Mean, color = Container, group = Container)) +
  geom_line() + geom_point(size = 3) +
  facet_wrap(~Light, labeller = label_both) +
  labs(y = "Mean Melting Time (min)", title = "Interaction Plot") +
  theme_bw()
```



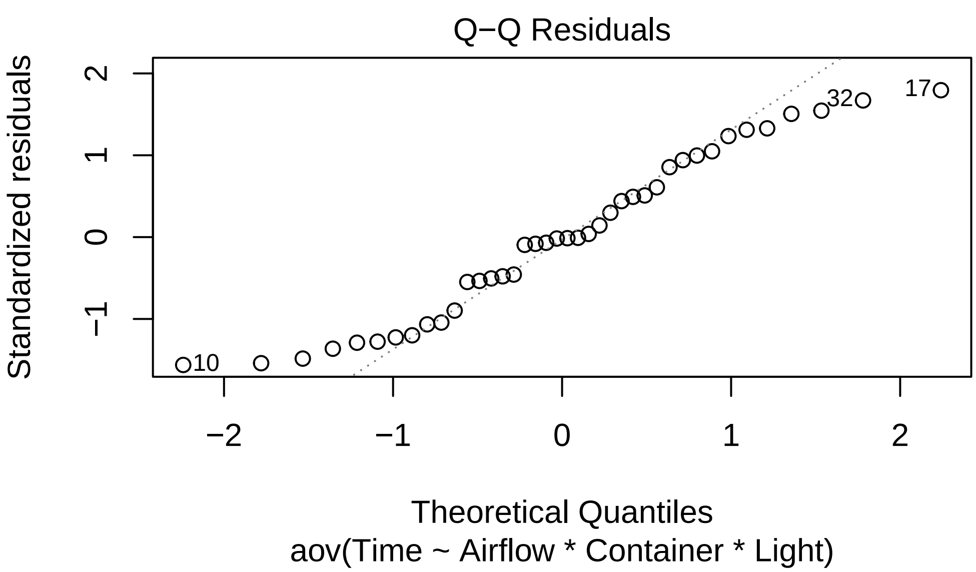
The interaction plot indicates that the plastic and metal lines are approximately parallel to each other for each light level, indicating little interaction between light and material type. Nevertheless, the difference between plastic and metal seems to be a bit greater for high light levels than low light levels, hinting at the possibility of three-way interactions. In any case, all lines indicate a decrease from low to high airflow, reiterating that higher airflow reduces melting time regardless of the material or light level

- **Hypothesis Testing**: Test your hypothesis with an appropriate statistical test (e.g., ANOVA).

We will employ a full factorial ANOVA design to examine the interaction effects of airflow, material of container, and exposure to light on the melting time. Prior to interpreting the outcome of the analysis, we will check the assumptions of standard ANOVA, which include normality of residuals and homogeneity of variance, using the Shapiro-Wilk test and Levene's test, respectively. The assumption of independence will be considered to hold, as the order of treatment in the runs was fully randomized.

Normality Assumption

```r
model <- aov(Time ~ Airflow * Container * Light, data = data)

plot(model, which = 2)
```



The Q-Q plot of standardized residuals follows the theoretical diagonal closely with only minor deviations at the tails, suggesting that the normality assumption is reasonably satisfied

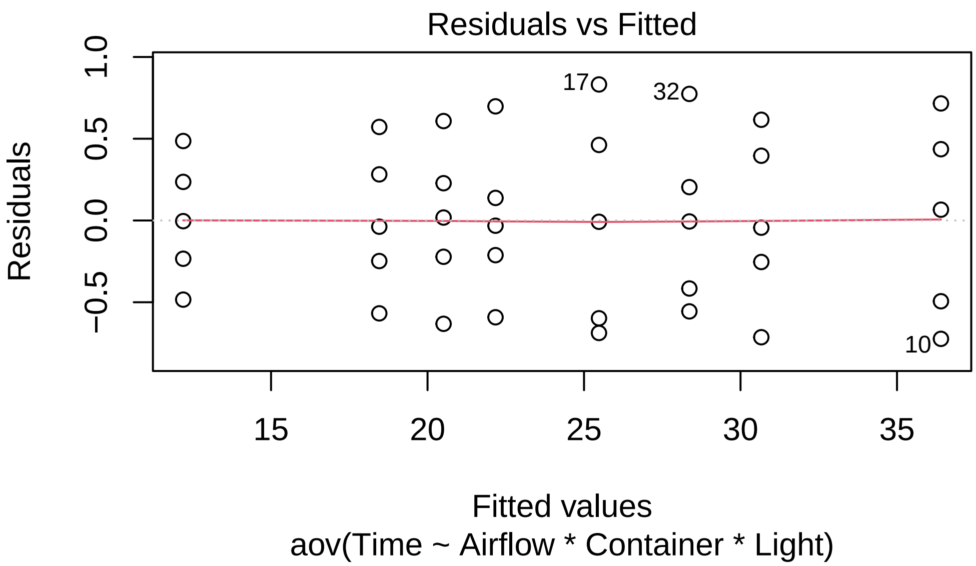
```r
shapiro.test(residuals(model))
```

```

	Shapiro-Wilk normality test

data:  residuals(model)
W = 0.9501, p-value = 0.07642
```

The Shapiro-Wilk test outputted $W = 0.9501$ and p-value $= 0.076$, which is above the 0.05 significance level, so we fail to reject the null hypothesis of normality and conclude that the normality assumption is satisfied.

Homogeneity of variance - residuals vs fitted

```r
plot(model, which = 1)
```



The residuals vs fitted plot shows no clear pattern or funneling, with residuals scattered roughly evenly around zero across all fitted values, suggesting that the homogeneity of variance assumption is satisfied.

```r
library(car)
leveneTest(Time ~ Airflow * Container * Light, data = data)
```

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  7  0.3811 0.9066
      32
```

Levene's test outputted F = 0.3811 and p-value = 0.907, which is well above the 0.05 significance level, so we fail to reject the null hypothesis of equal variances and conclude that the homogeneity of variance assumption is satisfied.

Further, all three assumptions of ANOVA were met: the residuals were normally distributed according to both the Q-Q plot and the Shapiro-Wilk test, the variances were equal for all groups according to both the plot of residuals vs. fitted values and Levene's test, and independence was established by the randomization of the order of the runs.

ANOVA TEST

For each effect in the 2^3 factorial model, we test the null hypothesis that the effect has no influence on the melting time against the alternative hypothesis that it does. Mathematically, for each main effect and interaction term, the null hypothesis is H : the effect is zero vs the alternative hypothesis H : the effect is nonzero. We test all three main effects (airflow, container, light), all three two-way interactions (airflow × container, airflow × light, container × light), and the three-way interaction (airflow × container × light) at a significance level of = 0.05

```r
summary(model)
```

```
                  Df Sum Sq Mean Sq  F value Pr(>F)
Airflow            1 1048.8  1048.8 3905.165 <2e-16 ***
Container          1  634.7   634.7 2363.438 <2e-16 ***
Light              1  335.4   335.4 1248.710 <2e-16 ***
Airflow:Container  1    0.9     0.9    3.307 0.0784 .
Airflow:Light      1    0.3     0.3    1.153 0.2909
```

```
Container:Light           1    1.9     1.9    7.143 0.0117 *
Airflow:Container:Light  1    0.4     0.4    1.658 0.2071
Residuals                32    8.6     0.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the results obtained, it is evident that all three factors, namely, airflow, container material, and light exposure, were highly statistically significant, as the p-value for each was less than 2e-16. The only significant interaction term among the ones calculated was the container:light interaction, which had an F-value of 7.143 and a corresponding p-value of 0.0117. This implies that the effect of light exposure on the melting time depends on the container material used. The other two-way interactions, namely, airflow:container and airflow:light, as well as the three-way interaction, were not significant, implying that the effect of airflow is independent of the other two factors.

- **Tables, Figures, & Code**: Include key results (ANOVA table, post-test) and relevant `R` code excerpts where needed.

Because the ANOVA indicates that there are significant effects present but does not indicate which combinations of factor levels differ from each other, we use a Tukey HSD test to compare all possible pairs of means while maintaining a family-wise error rate of $= 0.05$.

```r
TukeyHSD(model)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Time ~ Airflow * Container * Light, data = data)

$Airflow
            diff       lwr      upr p adj
High-Low -10.241 -10.57481 -9.90719     0

$Container
                diff       lwr      upr p adj
Metal-Plastic -7.967 -8.30081 -7.63319     0

$Light
           diff       lwr      upr p adj
High-Low -5.791 -6.12481 -5.45719     0

$`Airflow:Container`
                            diff        lwr        upr p adj
High:Plastic-Low:Plastic -10.539 -11.166921  -9.911079     0
Low:Metal-Low:Plastic     -8.265  -8.892921  -7.637079     0
High:Metal-Low:Plastic   -18.208 -18.835921 -17.580079     0
Low:Metal-High:Plastic     2.274   1.646079   2.901921     0
High:Metal-High:Plastic   -7.669  -8.296921  -7.041079     0
High:Metal-Low:Metal      -9.943 -10.570921  -9.315079     0

$`Airflow:Light`
                      diff        lwr        upr p adj
High:Low-Low:Low   -10.417 -11.044921  -9.789079     0
Low:High-Low:Low    -5.967  -6.594921  -5.339079     0
High:High-Low:Low  -16.032 -16.659921 -15.404079     0
Low:High-High:Low    4.450   3.822079   5.077921     0
High:High-High:Low  -5.615  -6.242921  -4.987079     0
High:High-Low:High -10.065 -10.692921  -9.437079     0
```

```
$`Container:Light`
                            diff        lwr        upr p adj
Metal:Low-Plastic:Low     -7.529  -8.156921  -6.901079     0
Plastic:High-Plastic:Low  -5.353  -5.980921  -4.725079     0
Metal:High-Plastic:Low   -13.758 -14.385921 -13.130079     0
Plastic:High-Metal:Low     2.176   1.548079   2.803921     0
Metal:High-Metal:Low      -6.229  -6.856921  -5.601079     0
Metal:High-Plastic:High   -8.405  -9.032921  -7.777079     0

$`Airflow:Container:Light`
                                      diff         lwr        upr     p adj
High:Plastic:Low-Low:Plastic:Low   -10.926 -11.9877045  -9.864296 0.0000000
Low:Metal:Low-Low:Plastic:Low       -8.038  -9.0997045  -6.976296 0.0000000
High:Metal:Low-Low:Plastic:Low     -17.946 -19.0077045 -16.884296 0.0000000
Low:Plastic:High-Low:Plastic:Low    -5.740  -6.8017045  -4.678296 0.0000000
High:Plastic:High-Low:Plastic:Low  -15.892 -16.9537045 -14.830296 0.0000000
Low:Metal:High-Low:Plastic:Low     -14.232 -15.2937045 -13.170296 0.0000000
High:Metal:High-Low:Plastic:Low    -24.210 -25.2717045 -23.148296 0.0000000
Low:Metal:Low-High:Plastic:Low       2.888   1.8262955   3.949704 0.0000000
High:Metal:Low-High:Plastic:Low     -7.020  -8.0817045  -5.958296 0.0000000
Low:Plastic:High-High:Plastic:Low    5.186   4.1242955   6.247704 0.0000000
High:Plastic:High-High:Plastic:Low  -4.966  -6.0277045  -3.904296 0.0000000
Low:Metal:High-High:Plastic:Low     -3.306  -4.3677045  -2.244296 0.0000000
High:Metal:High-High:Plastic:Low   -13.284 -14.3457045 -12.222296 0.0000000
High:Metal:Low-Low:Metal:Low        -9.908 -10.9697045  -8.846296 0.0000000
Low:Plastic:High-Low:Metal:Low       2.298   1.2362955   3.359704 0.0000016
High:Plastic:High-Low:Metal:Low     -7.854  -8.9157045  -6.792296 0.0000000
Low:Metal:High-Low:Metal:Low        -6.194  -7.2557045  -5.132296 0.0000000
High:Metal:High-Low:Metal:Low      -16.172 -17.2337045 -15.110296 0.0000000
Low:Plastic:High-High:Metal:Low     12.206  11.1442955  13.267704 0.0000000
High:Plastic:High-High:Metal:Low     2.054   0.9922955   3.115704 0.0000129
Low:Metal:High-High:Metal:Low        3.714   2.6522955   4.775704 0.0000000
High:Metal:High-High:Metal:Low      -6.264  -7.3257045  -5.202296 0.0000000
High:Plastic:High-Low:Plastic:High -10.152 -11.2137045  -9.090296 0.0000000
Low:Metal:High-Low:Plastic:High     -8.492  -9.5537045  -7.430296 0.0000000
High:Metal:High-Low:Plastic:High   -18.470 -19.5317045 -17.408296 0.0000000
Low:Metal:High-High:Plastic:High     1.660   0.5982955   2.721704 0.0003963
High:Metal:High-High:Plastic:High   -8.318  -9.3797045  -7.256296 0.0000000
High:Metal:High-Low:Metal:High      -9.978 -11.0397045  -8.916296 0.0000000
```

The Tukey HSD test for post-hoc analysis verified that all these comparisons were statistically significant at $= 0.05$. For the main effects, the result showed that high airflow had a reduction of 10.24 minutes in melting time compared to low airflow, metal containers had a reduction of 7.97 minutes in melting time compared to plastic containers, and high light exposure had a reduction of 5.79 minutes in melting time compared to low light. The three-way interaction analysis verified that the combination of high airflow, metal containers, and high light exposure had the fastest melting time, with a difference of 24.21 minutes compared to the baseline condition of low airflow, plastic containers, and low light, which had the largest difference in melting time compared to other combinations.

5 Conclusions

- Summarizing key findings.

In Conclusion, this experiment was conducted to examine the influence of airflow, container material, and light exposure on the melting time of ice cubes using a 2^3 factorial design. The findings have shown that all three variables significantly influenced the melting time of ice cubes, with airflow being the most dominant factor, followed by container material, and then light exposure. The melting time was decreased by 10.24 minutes with high airflow, 7.97 minutes with metal containers compared to plastic containers, and 5.79 minutes with high light exposure. The interaction of all three high-level variables resulted in the shortest melting time of 12.19 minutes on average, which is nearly three times faster than the slowest combination of variables at 36.40 minutes. Among the interaction variables, only the Container:Light interaction was statistically significant, which indicated that the advantage of high light exposure is further enhanced by the use of a metal container, probably because of the better heat conductivity of metal that absorbs the radiative energy from the lamp more effectively.

- Comments on limitations and possible improvements.

Regarding limitations, the experiment was carried out over four different days, which, despite our best efforts to standardize the conditions, allows for the possibility of subtle differences in the environment, such as minute variations in room temperature or humidity, which may have affected melting time. Furthermore, our ice cubes were not entirely identical, as they were made from the same ice tray but may have varied slightly in mass due to natural variations in the freezing process. A further limitation is that we only used two levels for each factor, which makes it difficult for us to accurately model the relationship between each factor and melting time, for instance, whether the relationship between melting time and airflow speed is linear or some other function of speed. Suggestions for future improvements might include standardizing the mass of the ice cubes prior to each trial, as well as carrying out the experiment in a more controlled environment, such as a climate-controlled laboratory, and using more levels for each factor to better understand the dose-response relationship between each factor and the rate of melting.